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1978 J. Phys. A: Math. Gen. 11 L121

(http://iopscience.iop.org/0305-4470/11/5/007)

J. Phys. A: Math. Gen., Vol. 11, No. 5, 1978. Printed in Great Britain

LETTER TO THE EDITOR

Intensity correlations in resonance fluorescence with atomic number fluctuations

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Received 14 March 1978

Abstract. The experimental conditions under which the intensity correlation function from a single atom undergoing resonance fluorescence may be observed are studied. As was pointed out by Jakeman *et al*, even under optimum conditions the number of fluctuations in the atomic beam prevent a direct observation of photon antibunching under the subsequent normalisation scheme used by Kimble *et al*. We show however that a direct observation of photon antibunching is possible in principle by adopting alternative normalisation schemes of the intensity correlation function.

In a recent experiment Kimble *et al* (1977) have measured the second-order correlation function of the scattered light field in resonance fluorescence and report to have observed photon antibunching for the first time. The fact that the fluorescent light from a driven two-level atom would exhibit photon antibunching was first predicted by Carmichael and Walls (1976) whose predictions are in agreement with subsequent calculations of Kimble and Mandel (1976) and Cohen Tannoudji (1977). This effect is intimately connected with the quantum nature of the electromagnetic field and its observation would provide experimental evidence of an electromagnetic field which cannot be described classically (Glauber 1964). However, the analysis of their experimental data by Kimble *et al* (1977) has been criticised by Jakeman *et al* (1977) who claim the treatment of the background radiation is inadequate and further that the effect of fluctuations in the number of scattering atoms is omitted.

In view of the basic importance of this experiment it is appropriate to analyse in some detail the results which one would expect in a realistic situation where the number of atoms in the scattering volume fluctuates and the background noise also contributes. In this Letter we compute the second-order correlation function of the scattered field taking into account both atomic number fluctuations and the background noise.

In an atomic beam experiment it is clear that the number of atoms in the scattering volume will fluctuate in time. Let us assume that there are $N(t)$ atoms contributing independently to the fluorescent light at time t. The scattered field observed is then

$$E(t)=\sum_{i=1}^{N(t)} E_i(t)+\epsilon \tag{1}$$

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0305-4770/78/0005-9121$01.00

where $E_i(t)$ is the scattered field from the ith atom and ϵ is a time-independent term describing the scattered light from the background. For simplicity we assume first that the detection region A is smaller than an area of coherence A_c (Born and Wolf 1970). Thus we may omit the spatial phase factors in the above sum. This restriction will be removed later on. Decomposing the field as usual into positive and negative frequency parts the second-order correlation function of the scattered field is

$$G^{(2)}(\tau)=\langle E^-(t)E^-(t+\tau)E^+(t+\tau)E^+(t)\rangle$$
$$=\Big\langle \sum_{i=1}^{N(t)}\sum_{j=1}^{N(t+\tau)}\sum_{m=1}^{N(t+\tau)}\sum_{n=1}^{N(t)}(E_i^-(t)+\epsilon^-)(E_j^-(t+\tau)+\epsilon^-)$$
$$\times(E_m^+(t+\tau)+\epsilon^+)(E_n^+(t)+\epsilon^+)\Big\rangle. \tag{2}$$

We assume that for times τ of interest the number of atoms within the scattering volume remains constant (i.e. the atomic fluctuations occur on a much longer time scale than the field fluctuations):

$$N(t+\tau)=N(t)\equiv N. \tag{3}$$

Under the conditions of the experiment there are no correlations between the field from different atoms and $\langle \Sigma_i E_i(t)\rangle = 0$. It is then readily shown that

$$G^{(2)}(\tau)=NG_A^{(2)}(\tau)+N(N-1)[I_A^2+|G_A^{(1)}(\tau)|^2]+|\epsilon|^4+2I_AN|\epsilon|^2+2\,\mathrm{Re}(G_A^{(1)}(\tau))N|\epsilon|^2 \tag{4}$$

where

$$I_A=\langle E_i^-E_i^+\rangle \tag{5}$$

is the intensity from a single atom, and

$$G_A^{(1)}(\tau)=\langle E_i^-(\tau)E_i^+(0)\rangle \tag{6}$$

and

$$G_A^{(2)}(\tau)=\langle E_i^-(0)E_i^-(\tau)E_i^+(\tau)E_i^+(0)\rangle \tag{7}$$

are the single-atom first- and second-order correlation functions, respectively. Their analytic form is given explicitly in Carmichael and Walls (1976).

The normalised correlation function is obtained by dividing by $|G^{(1)}(0)|^2$ where

$$|G^{(1)}(0)|=NI+|\epsilon|^2. \tag{8}$$

However when accumulating data over a time $T\gg\tau$ one must average over the atomic number fluctuations. In an atomic beam the fluctuations in the number of atoms are well approximated by a Poisson distribution. The quantity measured in the experiment by Kimble *et al* is a subsequently normalised correlation function defined by

$$\overline{g_S^{(2)}}(\tau)=\frac{\overline{G^{(2)}}(\tau)}{|\overline{G^{(1)}(0)}|^2} \tag{9}$$

where $\overline{(\ )}$ represents the average over the Poisson distribution. From equations (4) and (8) using $\overline{N(N-1)}=\bar{N}^2$ for a Poisson distribution we arrive at the result

$$\overline{g_S^{(2)}}(\tau)=1+\frac{1}{(1+\delta/\bar{N})^2}\left(|g_A^{(1)}(\tau)|^2+\frac{2\delta}{\bar{N}}\,\mathrm{Re}(g_A^{(1)}(\tau))+\frac{g_A^{(2)}(\tau)}{\bar{N}}\right) \tag{10}$$

where $1/\delta = I_A/|\epsilon|^2$ is the signal to noise ratio for a single atom. If the observation region A is larger than a coherence area A_c, the correlation function must be modified in order to account for the spatial averaging of the signal. Following the treatment of Jakeman *et al* (1970) and Jakeman (1974) we obtain

$$\overline{g_S^{(2)}}(\tau) = 1 + \frac{1}{(1+\delta/\bar{N})^2}\left(|g_A^{(1)}(\tau)|^2 f(A) + \frac{2\delta}{\bar{N}}\,\mathrm{Re}(g_A^{(1)}(\tau)) f_D(A) + \frac{g_A^{(2)}(\tau)}{\bar{N}}\right). \tag{11}$$

$f(A)$ and $f_D(A)$ are complicated functions given explicitly in Jakeman (1974). For our purposes it is sufficient to know that they are equal to one for $A \ll A_c$ and go to zero as $1/A$ for $A \gg A_c$. The $g_A^{(1)}(\tau)$ terms may be Doppler broadened due to the transverse component of atomic velocity introduced by the optical detection system.

There are several points worth mentioning about this result.

(1) The first term inside the brackets is a heterodyne term† resulting from the beating of light from different atoms. The second term arises from the heterodyning of the fluorescent light with the background light. The antibunching effect is contained in the third term, the single-atom correlation function divided by $\bar{N}$, hence its contribution decreases as $1/\bar{N}$. In the limit of zero noise and large $\bar{N}$ we recover the result of the central limit theorem (Carmichael and Walls 1976). In this limit the antibunching is lost, thus it is clear that one requires a low-density atomic beam to observe photon antibunching. The transition from small to large $\bar{N}$ is displayed in figure 1 where we plot $g_S^{(2)}(\tau)$ for $\delta = 0$.

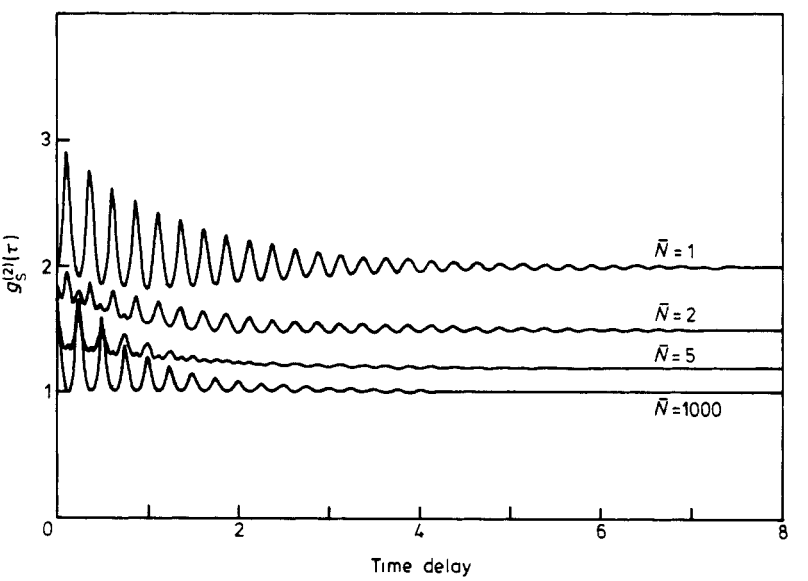

Figure 1. Subsequently normalised intensity correlation function for $\bar{N} = 1, 2, 5, 1000$, in the zero-noise limit. The delay time is in units of γ^{-1}, and the Rabi frequency is $\Omega = 25\gamma$.

(2) We note that for a sufficiently small number of scatterers the intensity correlation function does not go to 1 as $\tau \to \infty$. Rather there is a residual background due to the fluctuations in the number of scatterers. This effect has been known for some time and has been observed experimentally by Schaefer and Berne (1972).

† Alternatively this term may be seen as a beating of the signal with itself and called a homodyne term.

(3) Since $f(A)$ and $f_D(A)$ have approximately the same behaviour as a function of A/A_c, the first and second terms in the bracket in equation (11) are always present or absent simultaneously. In other words the heterodyne terms become negligible for the same value of A/A_c. The information presented in Kimble *et al* (1977) is insufficient to allow us to determine under what conditions their experiment was performed.

(4) It is evident that by using an observation volume $A \gg A_c$ one can eliminate these two heterodyne terms which serve to mask the antibunching effect. It is in this limit that the remarks of Jakeman *et al* (1977) are relevant. Since $g_A^{(2)}(0) = 0$ it is clear that $\overline{g_S^{(2)}}(0) = 1$. Thus, though a positive slope

$$\frac{d}{d\tau} g_S^{(2)}(\tau)|_{\tau=0} > 0$$

may be observed it is not possible to observe a $\overline{g_S^{(2)}}(0) < 1$. Thus the photon antibunching character of the light may only be inferred since it is superposed on the Poissonian number fluctuations of the atoms. This is somewhat disappointing as it appears it would be impossible to make a direct measurement of photon antibunching in resonance fluorescence.

We wish to suggest how this difficulty may be overcome by adopting alternative normalisations of the intensity correlation function $G^{(2)}(\tau)$. The result of these alternative normalisations is to reduce the effect of the atomic number fluctuations so that photon antibunching (i.e. $g^{(2)}(0) < 1$) may be directly observed†.

We turn now to a consideration of alternative normalisation schemes. We restrict ourselves to the situation $A \gg A_c$ so that the heterodyne terms are absent. Let us first consider an ideal situation where we have a fixed total number of scatterers N. Then the normalised second-order correlation function is

$$g_N^{(2)}(\tau) = \frac{G^{(2)}(\tau)}{|G^{(1)}(0)|^2} = 1 + \frac{(g_A^{(2)}(\tau) - 1)N}{(N+\delta)^2} \tag{12}$$

where we have used equations (4) and (8). Now consider the situation where we have a Poisson distribution of atoms in an atomic beam but measure the above normalised average during each transit time of the atoms. We must then average this independently normalised correlation function (Oliver 1974) over the Poisson distribution of the atoms. This yields

$$\overline{g_I^{(2)}}(\tau) = 1 + (g_A^{(2)}(\tau) - 1)\left(\overline{\frac{N}{(N+\delta)^2}}\right). \tag{13}$$

This function possesses the advantage that the contribution due to atomic number fluctuations is considerably reduced so that a $\overline{g_I^{(2)}}(0) < 1$ may in principle be observed. For example, in the low-noise case ($\lim \delta \to 0$), we obtain for $\bar{N} = 1$

$$\overline{g_I^{(2)}}(\tau) \simeq 1 + 0{\cdot}5(g_A^{(2)}(\tau) - 1). \tag{14}$$

A plot of $\overline{g_I^{(2)}}(0)$ against $\bar{N}$ for $\delta = 0{\cdot}25$ is shown in figure 2. This does not exhaust the ways we may take the normalised averages. It is also possible in principle to take a

† Alternatively it may be possible to reduce the effect of the fluctuations in the atomic number distribution by selecting the input data using a device such as a hot wire detector which provides information on the number of atoms present.

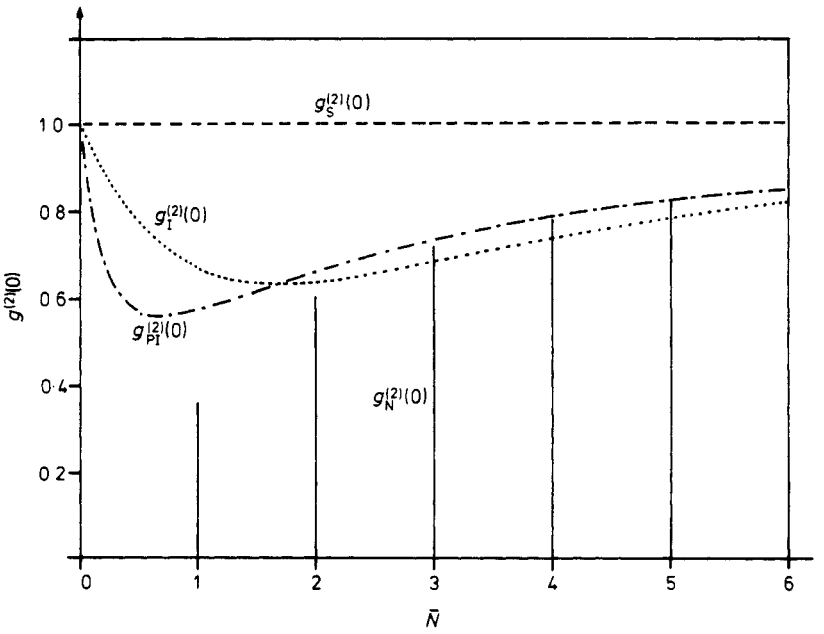

Figure 2. Second-order correlation functions $g_S^{(2)}(0)$ (subsequent normalisation), $g_I^{(2)}(0)$ (independent normalisation), $g_{PI}^{(2)}(0)$ (partial independent normalisation), and $g_N^{(2)}(0)$ (fixed number of scatterers), as a function of $\bar{N}$, for a noise $\delta = 0{\cdot}25$.

partially independently normalised correlation function as follows:

$$\overline{g_{PI}^{(2)}}(\tau) = \left(\frac{\overline{G^{(2)}(\tau)}}{|G^{(1)}(0)|}\right)\frac{1}{\overline{|G^{(1)}(0)|}}. \tag{15}$$

That is, the second-order correlation function is partially normalised by $|G^{(1)}(0)|$ during each transit time. This function has the form

$$\overline{g_{PI}^{(2)}}(\tau) = 1 + (g_A^{(2)}(\tau) - 1)\left(\frac{\overline{N}}{N+\delta}\right)\frac{1}{(\delta+\bar{N})}. \tag{16}$$

Again this quantity has the property of reducing the contribution from the atomic number fluctuations. Indeed, it is superior in this respect to $\overline{g_I^{(2)}}(\tau)$ as is shown in the plot of $\overline{g_{PI}^{(2)}}(0)$ in figure 2. In the low-noise case (lim $\delta \to 0$), we obtain for $\bar{N} = 1$

$$\overline{g_{PI}^{(2)}}(\tau) = 1 + 0{\cdot}632(g_A^{(2)}(\tau) - 1). \tag{17}$$

Note also that independent normalisation offers the further advantage of removing some of the effects of laser drift.

For comparison the subsequently normalised correlation function at $\tau = 0$ ($\overline{g_S^{(2)}}(0)$) and $g_N^{(2)}(0)$ for a fixed number N of scatterers are also plotted in figure 2.

In conclusion we have shown that the optimum experimental conditions to observe photon antibunching are such that the observation region is much larger than an area of coherence and the number of atoms in the observation region is very small (preferably $\bar{N} \lesssim 1$). However as was pointed out by Jakeman *et al* (1977) even under these optimum conditions the effect of the atomic number fluctuations is such as to negate the possibility of observing a $g^{(2)}(0) < 1$ under the subsequent normalisation

scheme used by Kimble *et al* (1977). However we wish to draw attention to the possibility of measuring independently normalised correlation functions which reduce the effect of the atomic number fluctuations and in principle enable a direct observation of photon antibunching ($g^{(2)}(0) < 1$) to be made.

DFW wishes to thank Professor H Walther for his hospitality at the University of Munich, and also wishes to acknowledge informative discussions with Drs Jakeman, Pike, Pusey and Vaughan at the Royal Signals and Radar Establishment.

Note added in proof. A formula equivalent to equation (1'1) has recently been derived by Kimble, Dagenais and Mandel (private communication) who have found excellent agreement with experiment.

References

Born M and Wolf E 1979 *Principles of Optics* (Oxford: Pergamon)
Carmichael H J and Walls D F 1976 *J. Phys. B: Atom. Molec. Phys.* **9** L43, 1199
Cohen Tannoudji C 1977 *Frontiers in Laser Spectroscopy* eds R Balcan, S Haroche and S Liberman (Amsterdam: North-Holland)
Glauber R J 1964 *Quantum Optics and Electronics* ed. C de Witt (New York: Gordon and Breach)
Jakeman E 1974 *Photon Correlations and Light Beating Spectroscopy* eds H Z Cummins and E R Pike (New York: Plenum)
Jakeman E, Oliver C J and Pike E R 1970 *J. Phys. A: Gen. Phys.* **3** L45
Jakeman E, Pike E R, Pusey P N and Vaughan J M 1977 *J. Phys. A: Math. Gen.* **10** L257
Kimble H J and Mandel L 1976 *Phys. Rev.* A **13** 2123
Kimble H J, Dagenais M and Mandel L 1977 *Phys. Rev. Lett.* **39** 691
Oliver C J 1974 *Photon Correlations and Light Beating Spectroscopy* eds H Z Cummins and E R Pike (New York: Plenum)
Schaefer D W and Berne B J 1972 *Phys. Rev. Lett.* **28** 475